

Mathematics of Everything

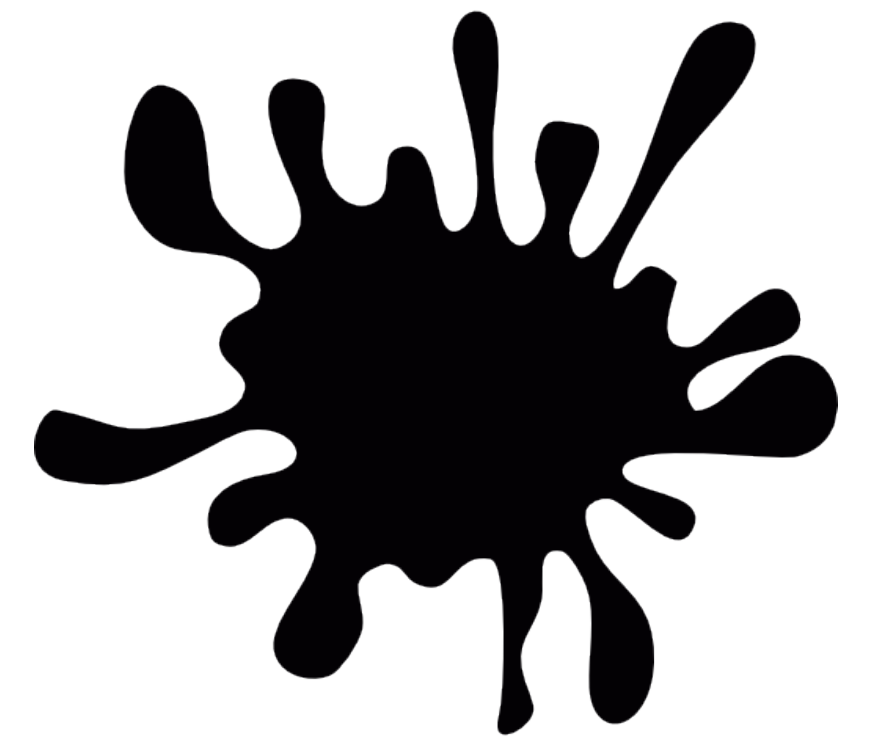
Gadadhar Misra, Indian Institute of Science
December 18, 2020



Who



Where



What

Mathematics of Everything



Who



Where



What

Who am I?



Ask Google!

Google: The index of the internet



"They're encyclopedias, Timmy. . . they're an early form of **Google**."

How does 9009LE work?

Google, the indispensable tool of the internet era, is driven by non-trivial mathematics behind its workings.

The essential idea comes from a theorem of Frobenius and Perron dealing with Markov chains.

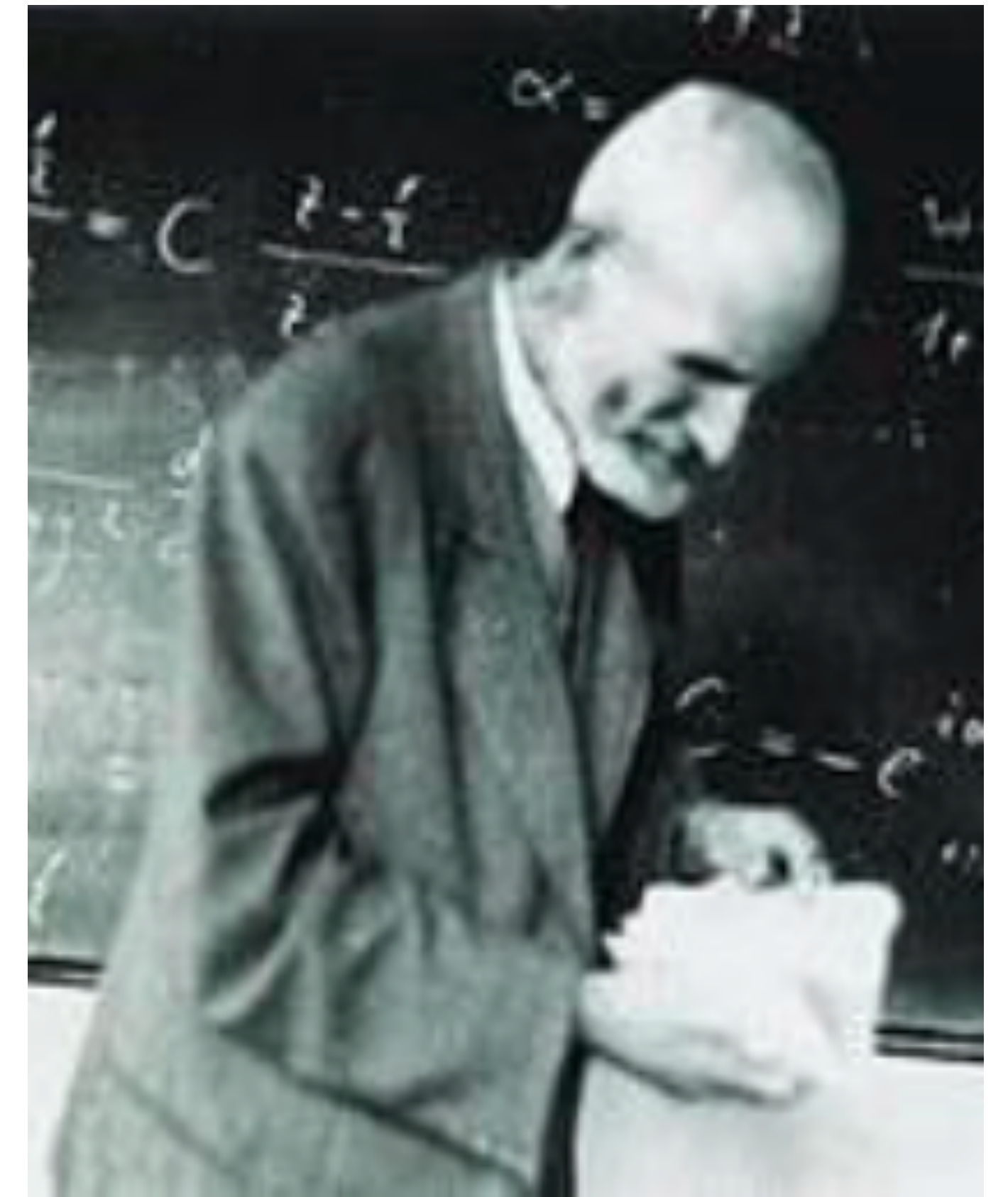
The name Google is a misspelling of [Googol](#), the number 1 followed by 100 zeros



Georg Frobenius
(1849 — 1917)



A. A. Markov
(1856 — 1922)



O. Perron
(1880 — 1975)

The problem of searching

What we have at hand is
a **query** (say Q) and a set of **webpages** (say W).

This is a problem in two parts:

- a) determine which webpages in W “**match**” the query Q , and,
- b) prioritise these results in **ranking from most to least “relevant”**.

About 47,40,00,000 results (0.91 seconds)

en.wikipedia.org › wiki › Michael_Douglas

Michael Douglas - Wikipedia

Michael Kirk Douglas (born September 25, 1944) is an American actor and producer. He has received numerous accolades, including two Academy Awards, five Golden Globe Awards, a Primetime Emmy Award, the Cecil B. DeMille Award, and the AFI Life Achievement Award.

Political party: Democratic Nationality: American
Years active: 1966–present Education: Allen-Stevenson School; Choat...

Michael Douglas on stage and ... · Catherine Zeta-Jones · Cameron Douglas

People also ask

- What disease does Michael Douglas have? ▾
- Does Michael Douglas have Parkinson's? ▾
- How much money is Michael Douglas Worth? ▾
- What happened with Michael Douglas? ▾

Feedback

www.imdb.com › name

Michael Douglas - IMDb

Michael Douglas, Actor: Behind the Candelabra. An actor with over forty years of experience in theatre, film, and television, Michael Douglas branched out into ...

Star Sign: Libra Alternate Names: M.K. Douglas
Other Works: PSA for OralCancer.org Official Sites: Facebook | Instagram

Other Works · News · 1 of 976 · Photo Gallery

www.imdb.com › name › bio

Michael Douglas - Biography - IMDb



Michael Douglas

Actor

facebook.com/MichaelDouglasOfficial

Michael Kirk Douglas is an American actor and producer. He has received numerous accolades, including two Academy Awards, five Golden Globe Awards, a Primetime Emmy Award, the Cecil B. DeMille Award, and the AFI Life Achievement Award. [Wikipedia](#)

Born: 25 September 1944 (age 76 years), [New Brunswick, New Jersey, United States](#)



Where is the Physicist Michael Douglas?



Sergey Brin and Larry Page

Reasoning about a Highly Connected World

DAVID EASLEY

and

JON KLEINBERG

NETWORKS

CROWDS

AND MARKETS



CAMBRIDGE

The Web is a directed graph

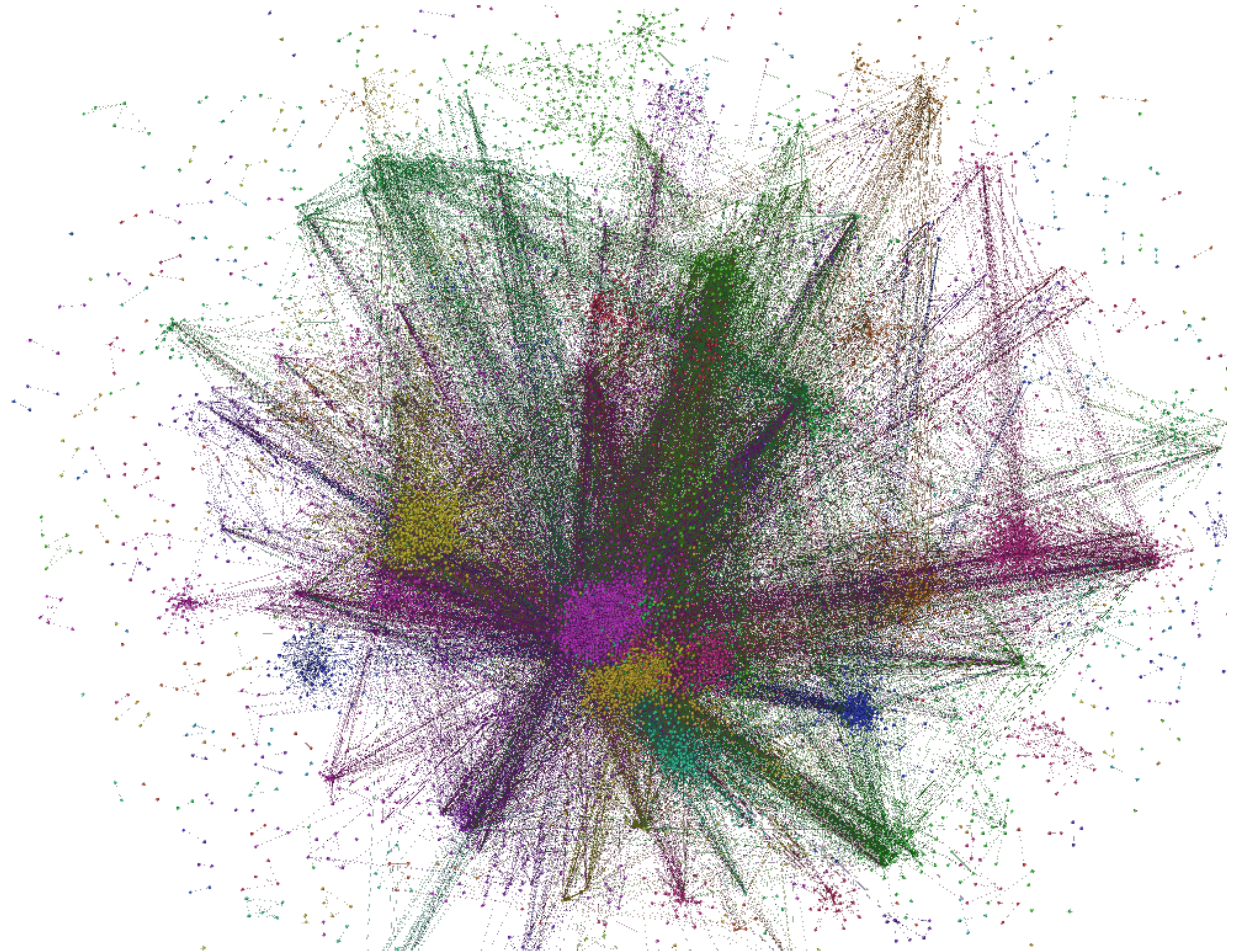
The nodes or vertices are the web pages.

The edges are the links coming into the page and going out of the page.

The Web Graph

T. Aynaud and P. Roseau,
Citero Labs

This graph has billions
of vertices and is
growing every
second!



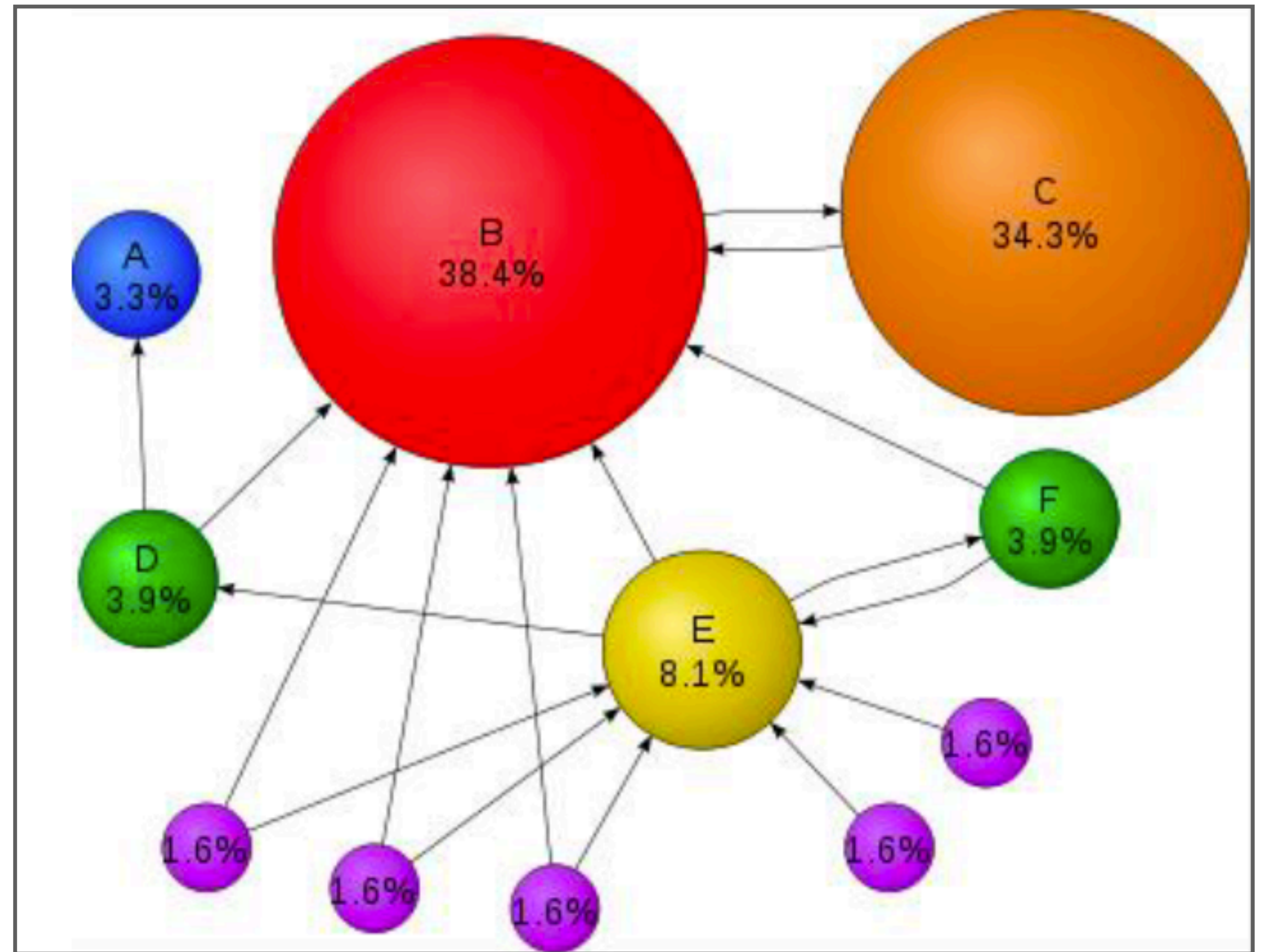
The PageRank Algorithm

PageRank Axiom

A webpage is important if it is pointed to by other important pages.

The algorithm was patented in 2001.

C has a higher rank than E, even though there are fewer links to C since the one link to C comes from an “important” page.



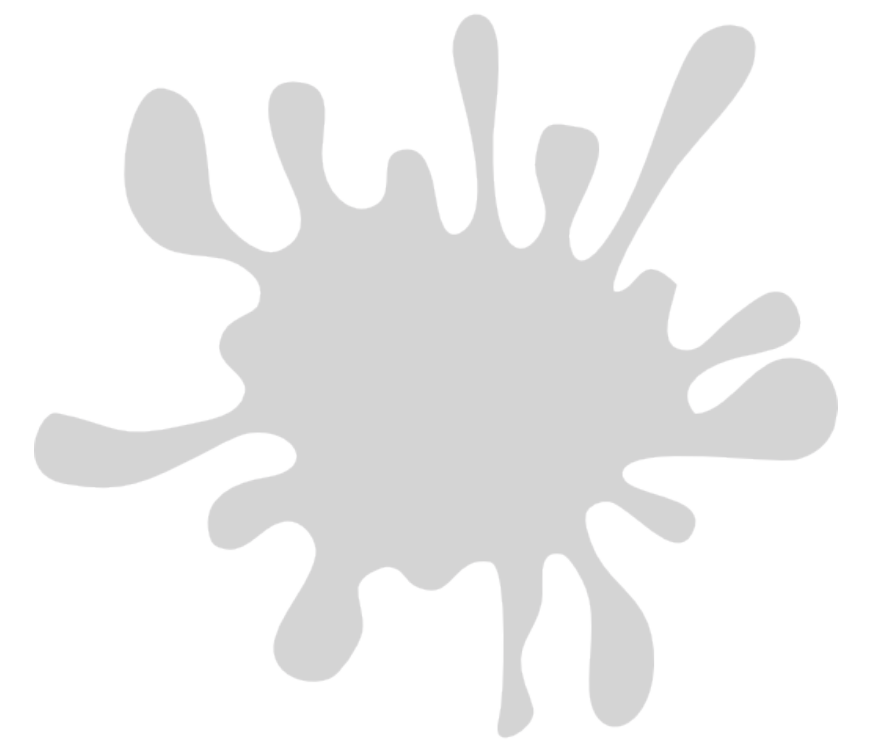
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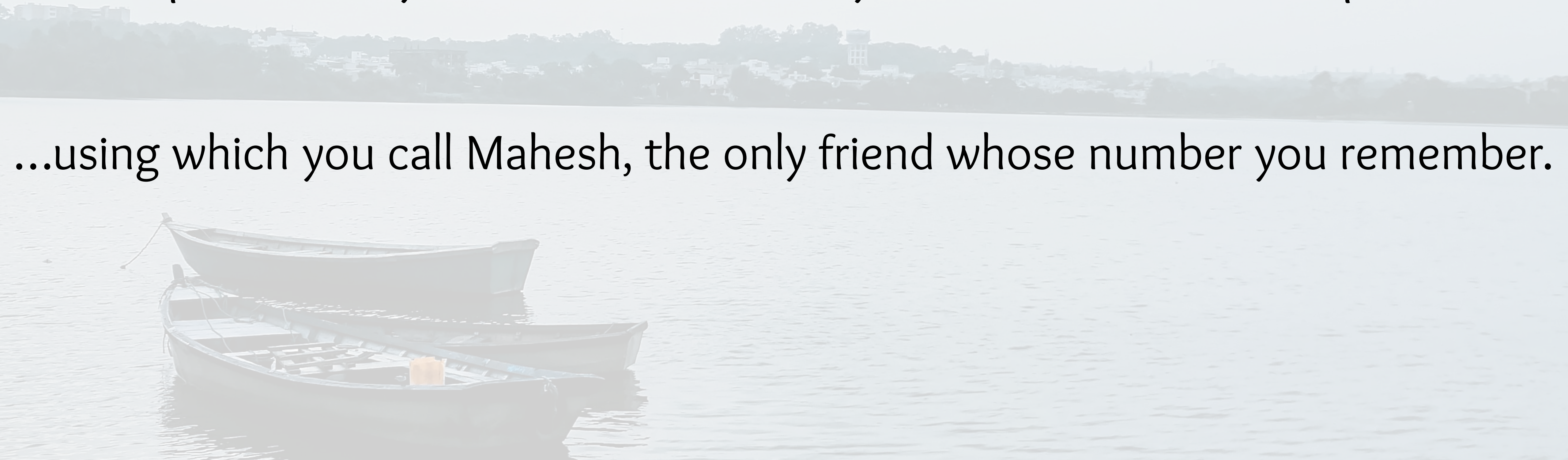


What

One day, you wake up in the middle of nowhere, with only a big India map, ruler, and compass. The first thing you want to do, naturally, is find out where you are located.

But, it is cold outside and you can't leave the room, and you don't have a cellphone. But you see that the room you are in has a landline phone...

...using which you call Mahesh, the only friend whose number you remember.



Mahesh says: “I’m in Gandhinagar and at a distance of 600KM”.



This is not enough information for you to figure out where you are though. So you ask for Anita’s number.

*It was a magic landline!

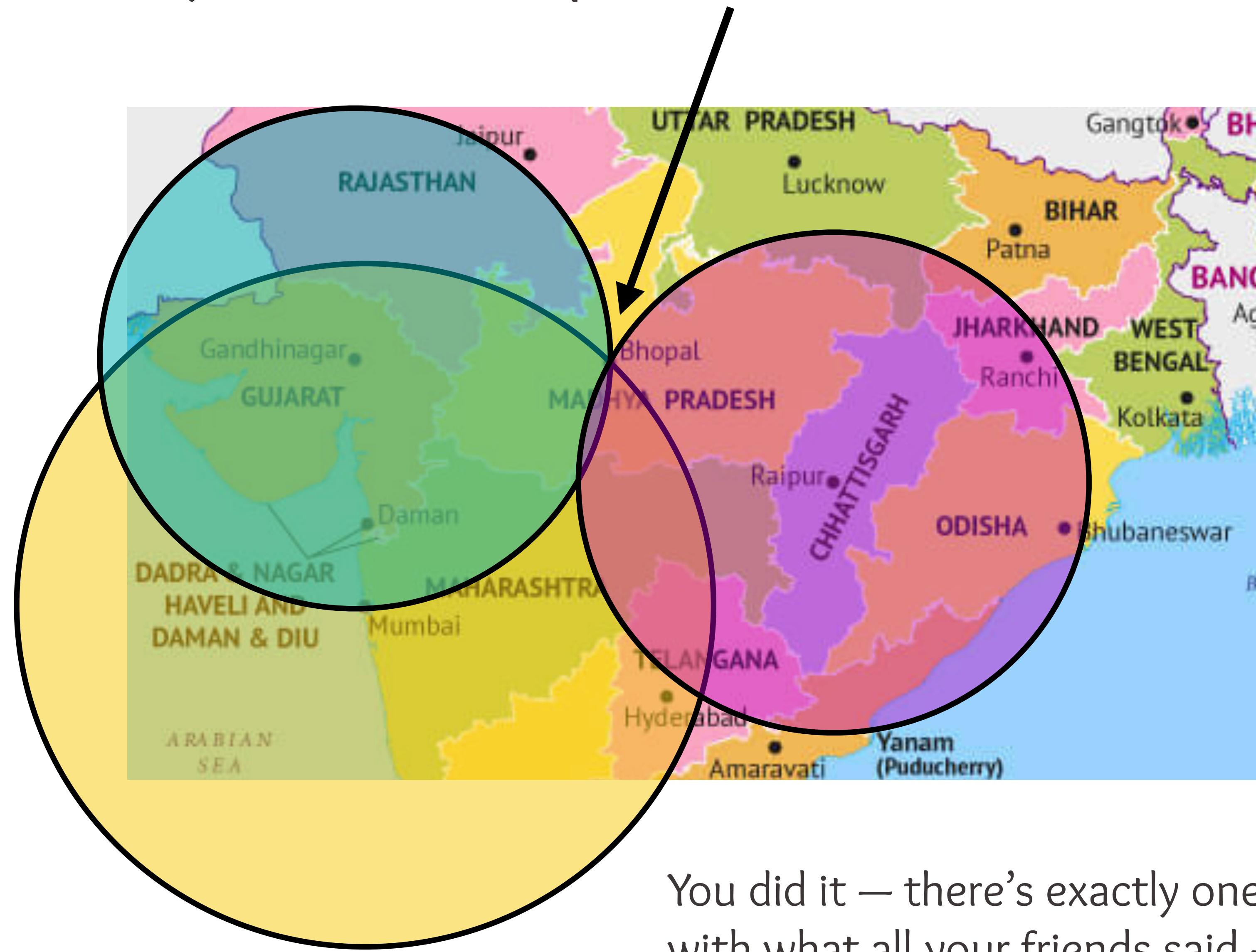
Anita says: “I’m in Mumbai and at a distance of 775KM”.



*It was a magic landline!

You could still be in many possible places, although this ruled out a few options. So you ask for Matthew’s number.

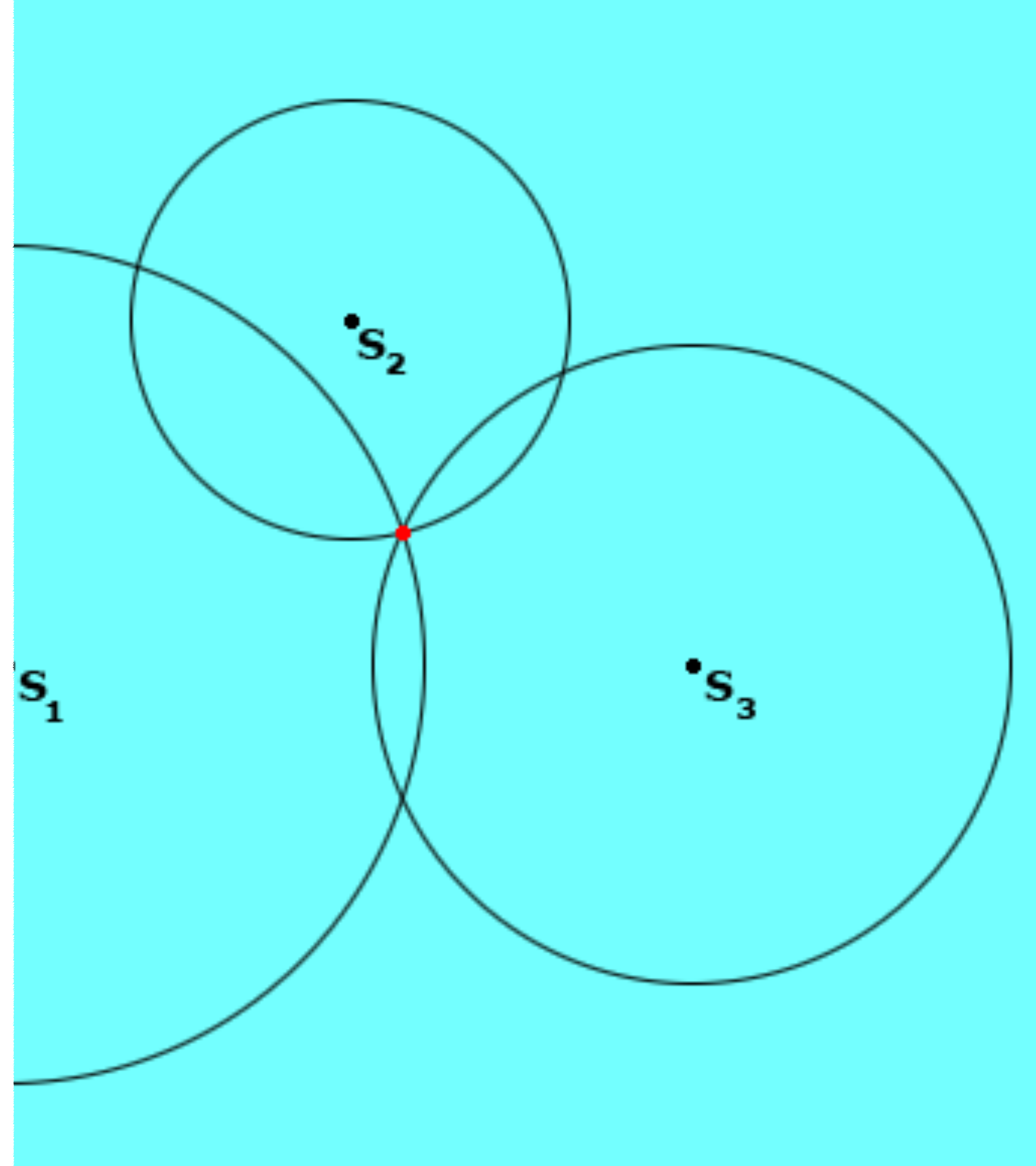
Matthew says: "I'm in Raipur and at a distance of 625KM".



You did it — there's exactly one point that's consistent with what all your friends said - and this is... **Bhopal!**

*It was a magic landline!

If you happen to know that you are at a distance x_1 from some place s_1 , distance x_2 from some other place s_2 and finally, at a distance x_3 from a place s_3 , then you can find your position.



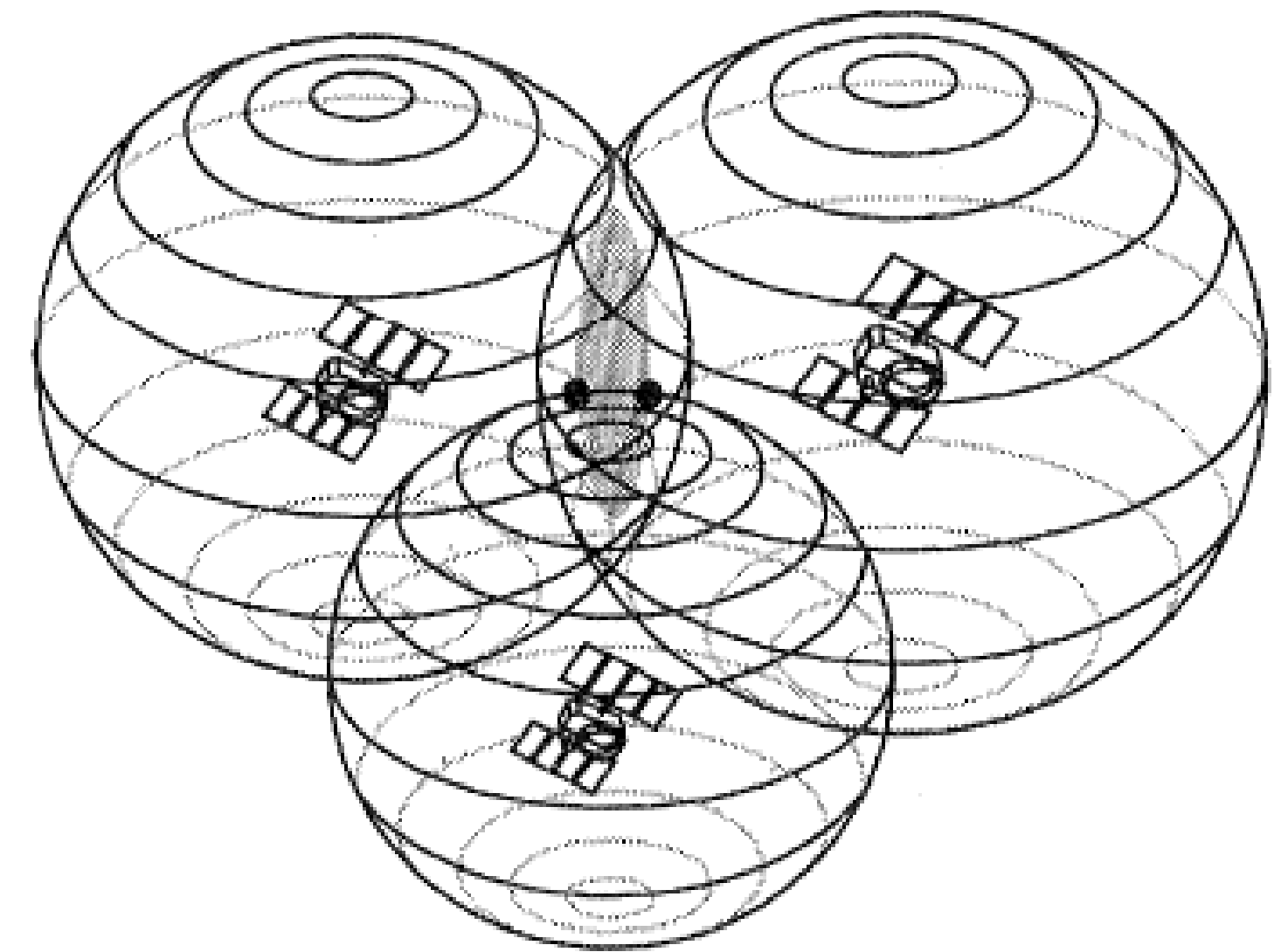
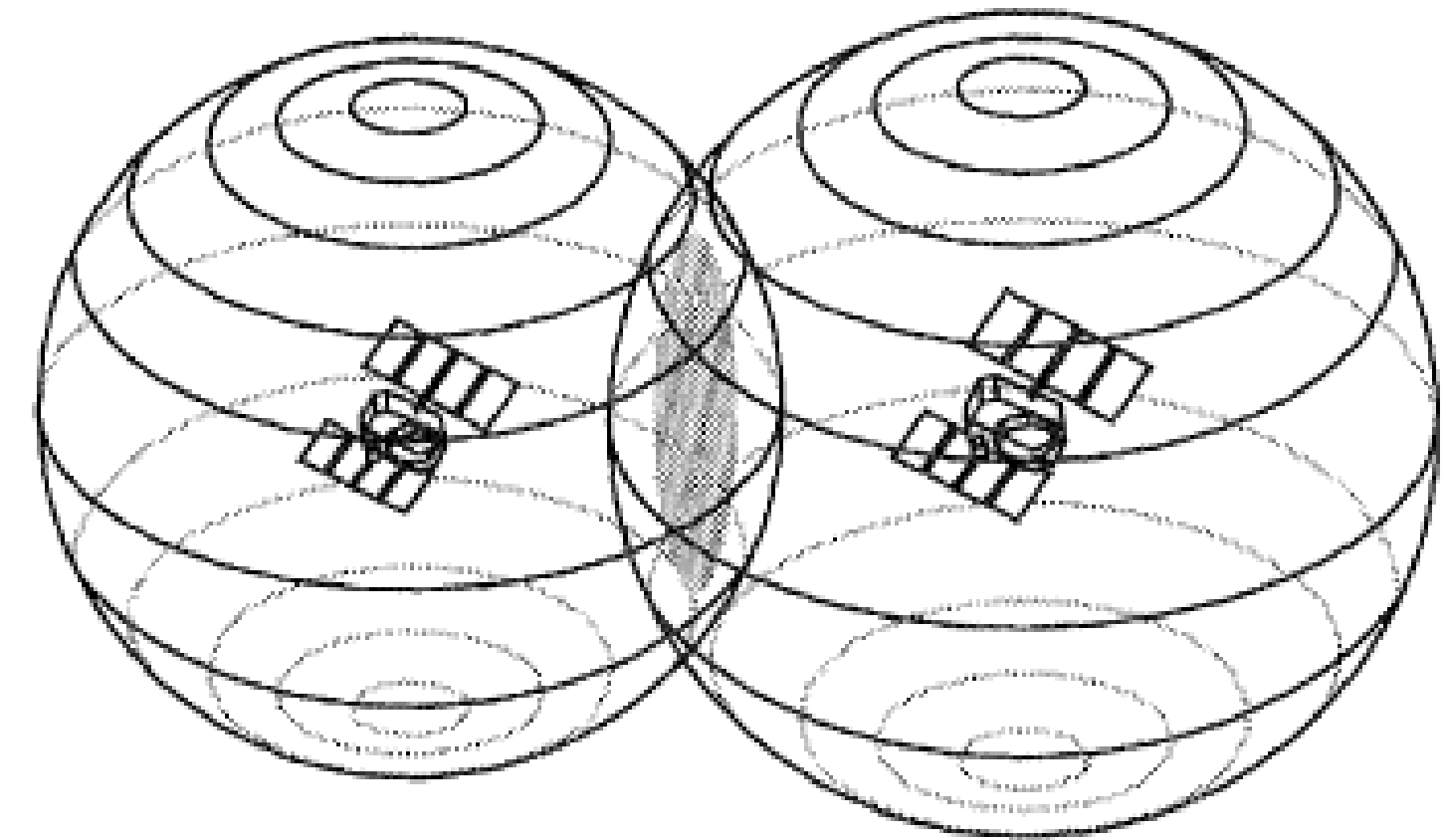
Spherical Geometry and a little bit of relativity!

Once you know three accurate distances from three specific locations, it is an easy matter to plot their intersection on a two dimensional map.

The intersection of two spheres is either empty or a circle.

The circle will intersect a third sphere in at most two points.

This geometric fact is the basis of GPS since other factors can be used to eliminate one of the two points as being an irrelevant solution to the problem.



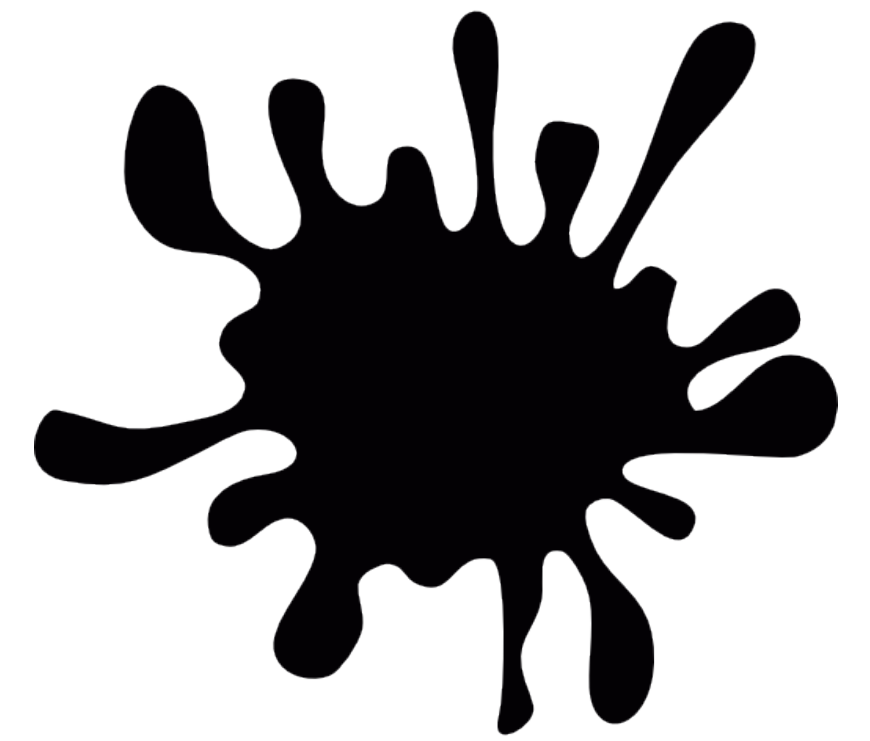
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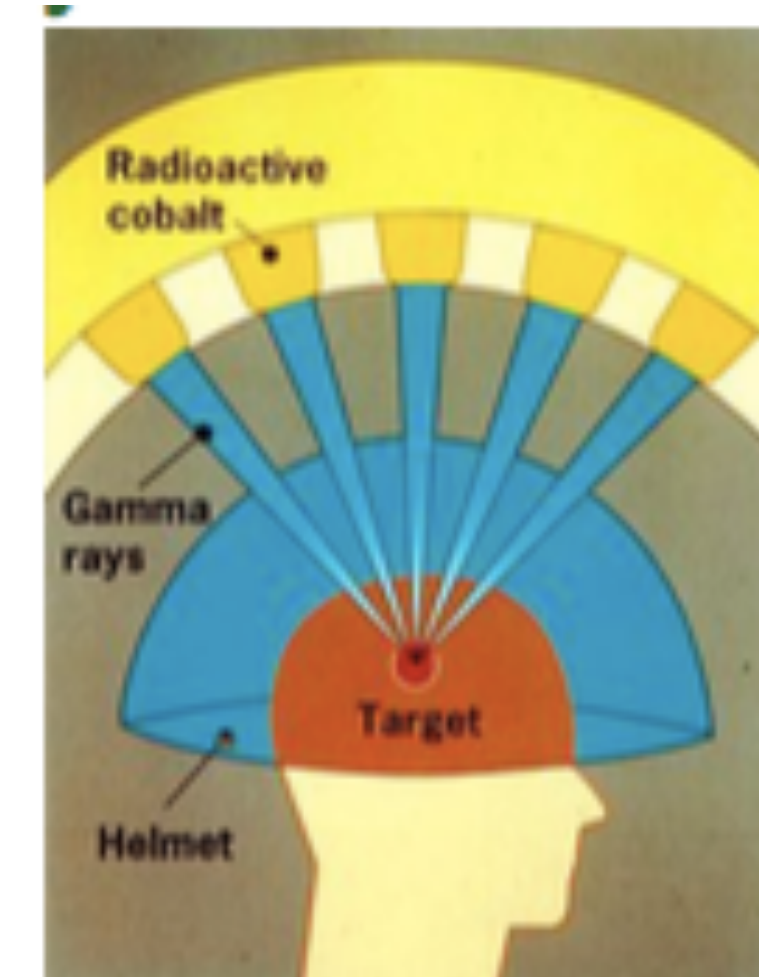
What is gamma-knife surgery?

It is a non-invasive medical procedure used to treat tumors, usually in the brain.

This is called radio surgery since it uses radiation to perform the surgery.

201 Cobalt gamma ray beams are arrayed in a hemisphere and aimed through a collimator to a common focal point.

The patient's head is positioned so that the tumor is the focal point.



The minimax problem

Since the tumor maybe of irregular shape and spread over a region, the idea is to minimize the number of radiation treatments and maximize the portion of the area to be treated.

When the beams are focused with the help of a helmet, they produce focal regions of various sizes.

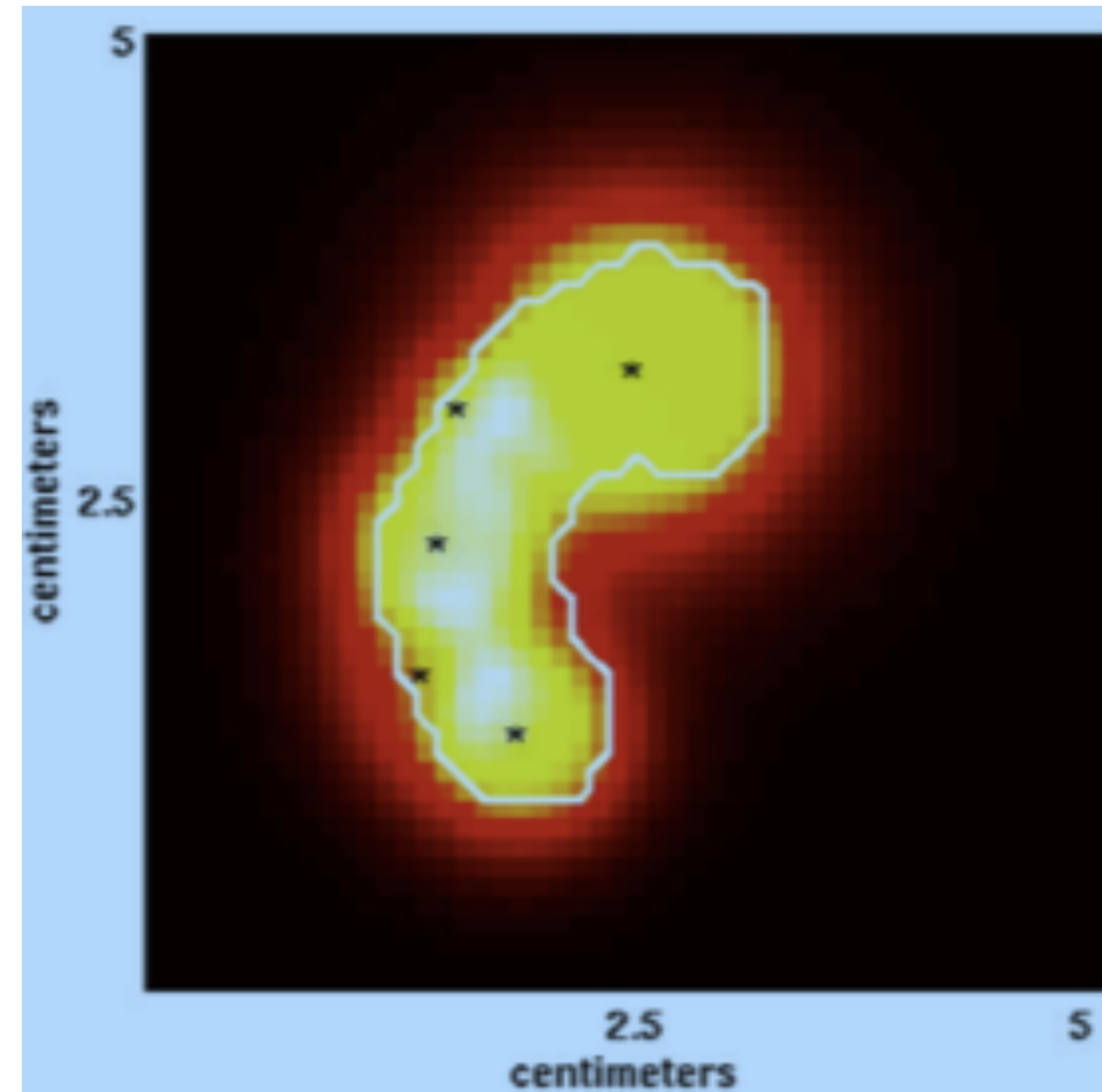
Each size of dose requires a different helmet and so the helmet needs to be changed when the dose radius needs to be changed.

Since each helmet weighs 500 pounds, it is important to minimize the number of helmet changes.

The mathematics of shapes

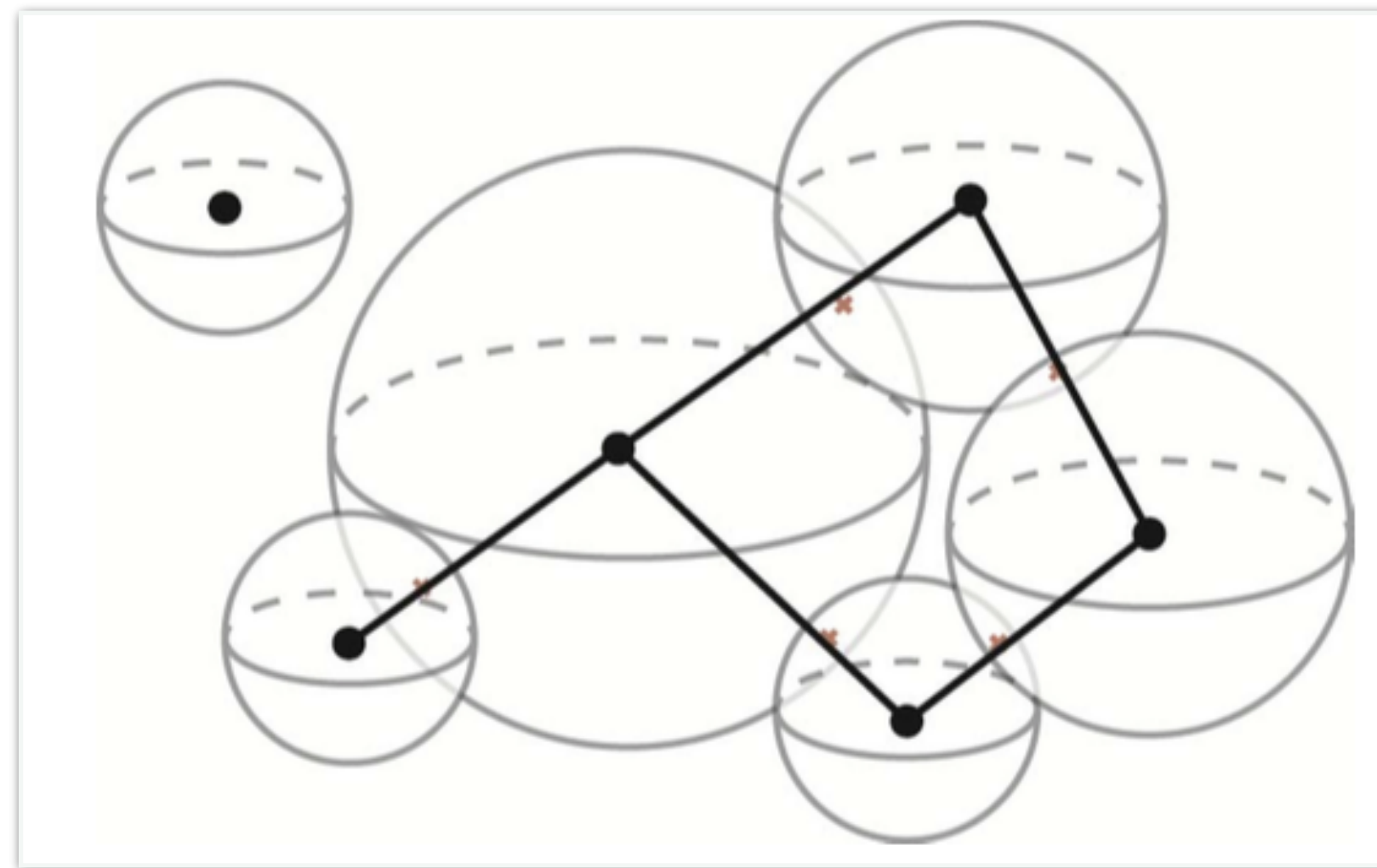
Here is the target area on which the radiation is to be applied.

Since the helmets have varying degrees of focal regions, several helmets have to be used.



Sphere packing problem

Since we have spheres of different sizes and not all of the affected region can be targeted, the problem can be formulated mathematically as follows:



The skeleton of a region

Let $|X-Y|$ denote the Euclidean distance between two points in the plane or in space.



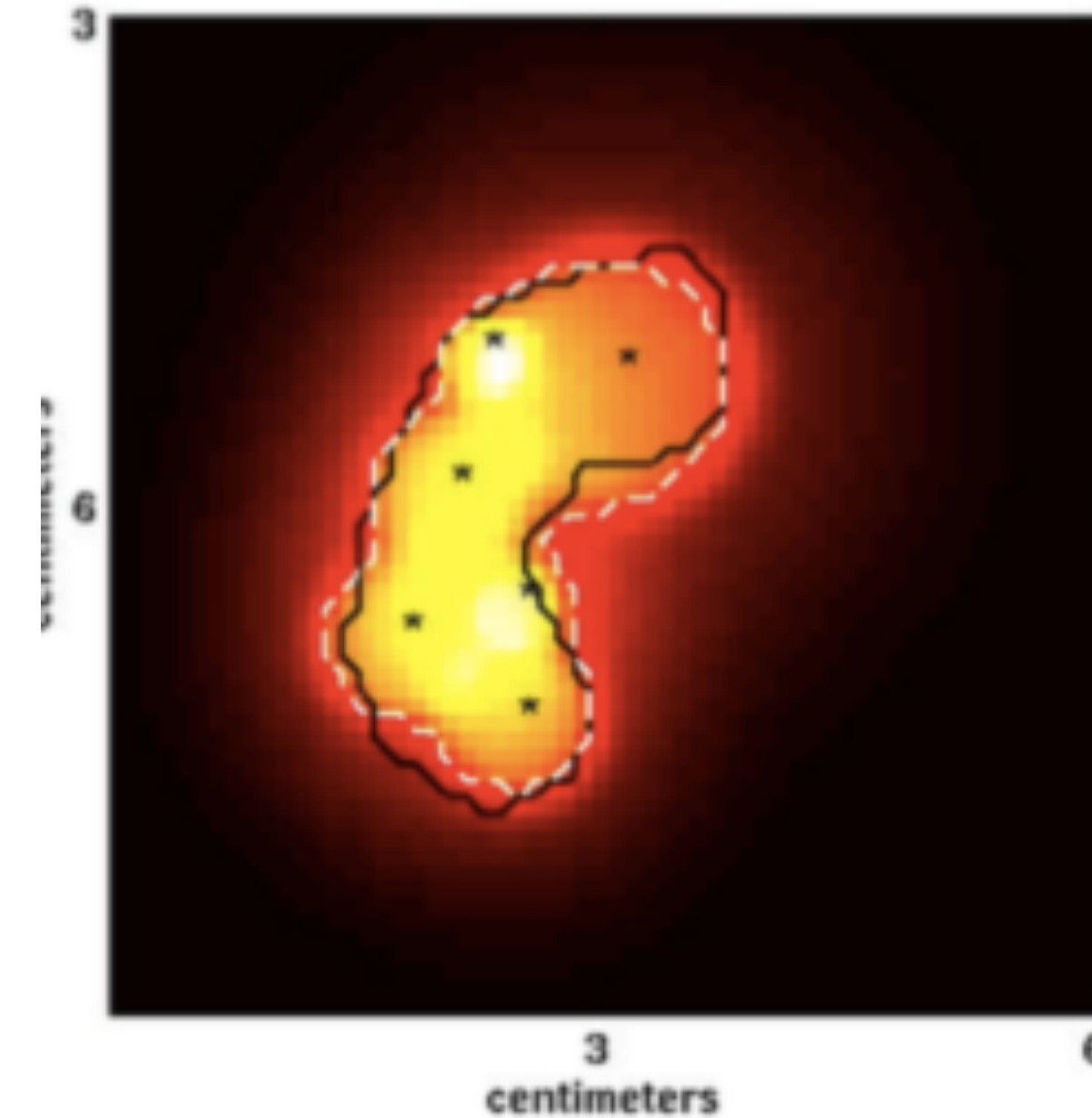
Fig. 4.1. The skeleton of a region.

The skeleton of a region R with boundary S is the set of points z for which there exist distinct points x, y on S such that

$$|z - x| = |z - y| = \min_{w \in S} |z - w|$$

Skeletons in R^3

The gamma rays will be focused on selected points along the skeleton of the region.

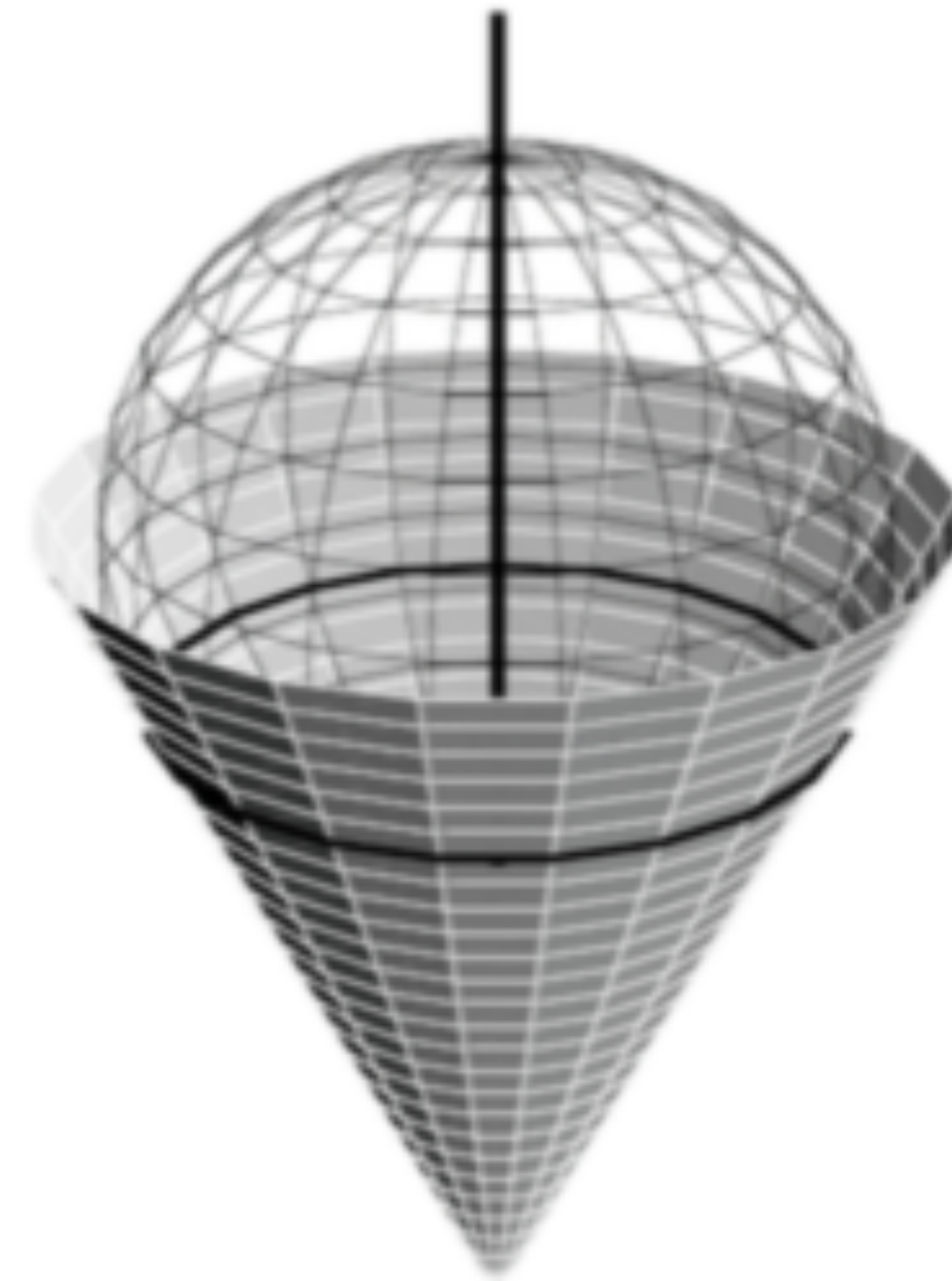


Three dimensional skeletons

Our earlier definition of a skeleton applies in higher dimensions as well, and in particular to R^3 . However, here we can distinguish two portions of the skeleton.

Some simple examples

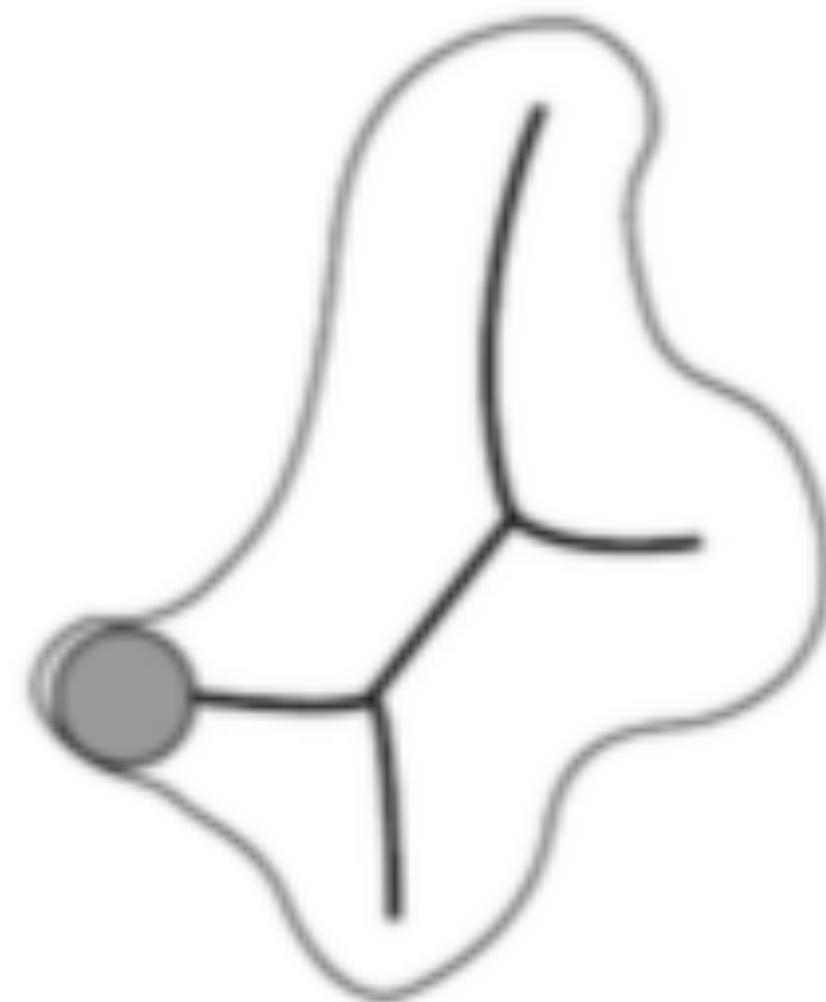
While the region is the solid filled cone, only the boundary is shown as well as one maximal ball and its circle of tangency.



(a) The skeleton of a solid circular cone is given by its central axis

The optimal surgery algorithm

Any dose in an optimal solution must be centered along the skeleton. If we have four sizes of doses, $a < b < c < d$ (say), then the initial doses should be at an extreme point of the skeleton.



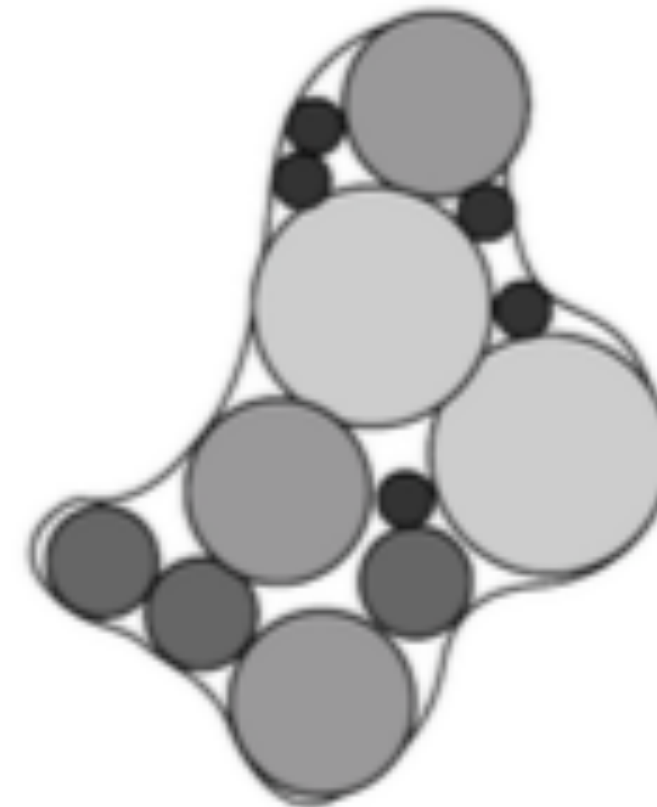
(a) A first dose of radius 4 mm

The iterative procedure

After the first dose, the region has changed and we need to re-calculate the skeleton.



(b) The skeleton of the remaining region after two doses of radii 4 and 7 mm



(c) The entire region irradiated with doses of radii 2, 4, 7 and 9 mm

Thank you!

For more details, please head over to:

<http://www.mast.queensu.ca/~murty/>

Icons are from  **flaticon**

Photograph by Shivam Tiwari  (unsplash.com).